Hale School

Mathematics Specialist

Test 5 --- Term 3 2019

Applications of Differentiation and Modelling Motion

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Name: ______________________

Instructions:

- **Calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Use exact values in your answers.

Question 1 **(3, 4 = 7 marks)**

(a) Determine $\frac{dy}{dx}$ for the relation $y\ln(x) = e^{2y} + 3x - 4$.

(b) Find the gradient of the curve with equation $2x^2 \sin(y) + xy = \frac{\pi^2}{18}$ at the point $\left(\frac{\pi}{6}, \frac{\pi}{6}\right)$.

Give your answer in the form $\frac{a}{\pi\sqrt{b} + c}$, where $a,\ b$ and c are integers.

Question 2 **(4 marks)**

Given the stated conditions, determine the general solution to the following differential equation:

$$\frac{dy}{dx} = \frac{3-y}{2}, \quad y \geq 3 \ .$$

Question 3 **(4 marks)**

The acceleration of a beam of light along a straight lamp post is given by the expression $a(t) = x - 7$ where $x(t)$ is in metres and t is in seconds; $v(0) = 7$ m/s , $x(0) = 0$.

Find v in terms of x .

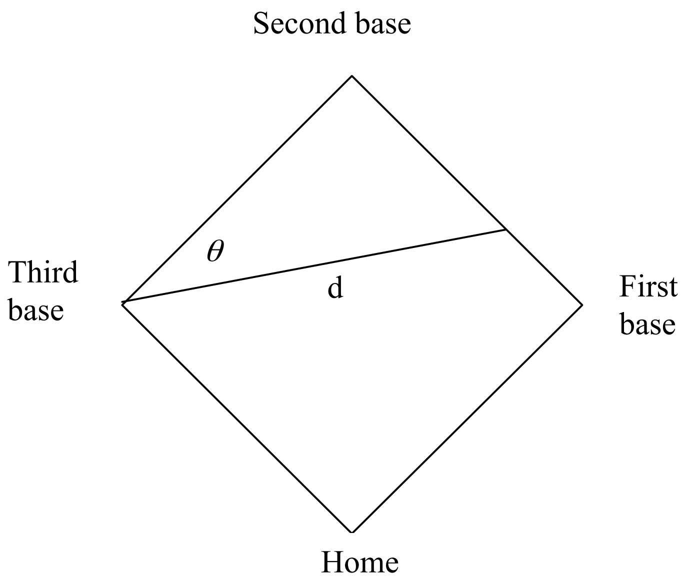
Question 4 **(6 marks)**

Use the separation of variables method to solve the following differential equation

$$\frac{dP}{dt} = 0.1P\left(1 - 0.05P\right)\,,\ P(0) = 1\ .$$

Question 5 **(4, 4 = 8 marks)**

A baseball diamond is a square, 30 metres on each side. A player runs from first base to second base at a rate of 5 metres per second.

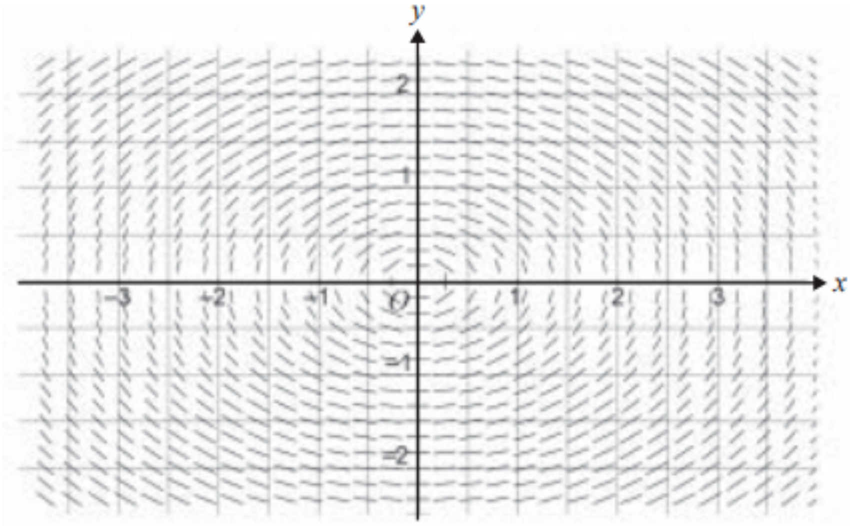


(a) At what rate is the player's distance from third base, d, changing when the player is 10 metres from second base?

(b) As the player slides into second base, the angle θ is changing at 8 degrees per second. Determine the speed of the player in metres per second at this instance.

Question 6 **(2, 2 = 4 marks)**

(a) The direction (slope) field for a certain first order differential equation is shown below.



Circle the differential equation that may respresent the slope field.

A. $\frac{dy}{dx} = \frac{x^2}{2} + y^2$

B. $\frac{dy}{dx} = x^2 + \frac{y^2}{2}$

C. $\frac{dy}{dx} = -\frac{x}{2y}$

D. $\frac{dy}{dx} = -\frac{y}{2x}$

E. $\frac{dy}{dx} = \frac{x}{2y}$

(b) For the differential equation $\frac{dy}{dx} = \frac{x}{2y}$ passing through the point (-1,1), use the incremental formula $\delta y = \frac{dy}{dx} \times \delta x$, with $\delta x = 0.1$ to calculate an estimate for the y - coordinate of the curve when $x = -1.1$.

Question 7 **(4, 5 = 9 marks)**

A particle is undergoing Simple Harmonic Motion such that $\dfrac{d^2x}{dt^2} + 4\pi^2 x = 0$

(a) Given that the particle begins at the origin with positive velocity, and has a maximum velocity of 8π m/sec, determine the displacement of the particle at any time *t*.

(b) Determine the acceleration of the particle when the particle first has negative displacement and a velocity of 4π m/sec.

__End of Test__